Sharpness
a Tight condition for Throughput Scalability

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Brief announcement @ PODC, 14th August 2007
Tip #1: Motivation

- Application deployed on a large distributed system (peer to peer networks, data stream processing, …)
- It is **throughput scalable** if the rate of computation and communication tasks is independent of the size.

- Challenge: What are the exact conditions of such scalability?
Tip #1: Motivation (cont’d)
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- Infinite tandem of single-server queue

  - Service of customer $m$ starts in station $k$ as soon as $m$ has arrived and $k$ is available:

  $$T(m, k) = S(m, k) + \max\left( T(m, k - 1), T(m - 1, k) \right).$$
Tip #1: Motivation (cont’d)

- Infinite tandem of single-server queue

\[ A_{m} \] \[ A_{m-1} \] \[ A_{m+1} \]

\[ k = 0 \quad k = 1 \quad k = 2 \]
Tip #1: Motivation (cont’d)
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- Infinite tandem of single-server queue
  - with finite buffer and **BLOCKING**

- New constraint (in addition to previous one):
  - Task T(m,k) is allowed only when T(m-B,k+1) is done.

- Throughput scalability still holds [Martin QUESTA 2002]
Tip #1: Motivation (cont’d)

- Infinite tandem of single-server queue

```
m + 1
A_{m+1}  ...

m
A_m  ...

m - 1
A_{m-1}  ...

...  ...

m - B
A_{m-B}  ...

...  ...

k = 0  k = 1  k = 2
```
Tip #2: The model used in this talk

- A directed graph:
  - vertices represent tasks to complete
  - edges represent dependencies

  Vertices set are $\mathcal{V} = \mathbb{Z}^d \times \mathcal{H}$ where $\mathcal{H}$ is finite.

  Collection of Edges $\mathcal{E}$ is invariant by translation.

- This model is analogous to Uniform Recurrence Equations [Karp et al. JACM67].
Tip #3: The main result

● Sharpness:
  - a dependence path \( \pi : a \times h \rightsquigarrow a' \times h' \) is called a dependence cycle if \( h' = h \). We define its associated vector as \( r = a' - a \).

  **Condition 1.** There exists \( s \) in \( \mathbb{Z}^d \), called a sharp vector, such that \( \langle r, s \rangle \leq -1 \) for all \( r \) associated with a dependence cycle.
Tip #3: The main result (cont’d)

- The system is throughput scalable if
  - the sharpness condition holds,
  - the weights satisfy the moment condition \( \int_0^\infty \mathbb{P}[\bar{s} \geq u]^{\frac{1}{\delta}} \, du < \infty \).

Sketch of the proof

- tasks completion time = last-passage percolation time
- sharpness implies that dependence paths have the same combinatorial properties as lattice’s connected subsets.

- Sharpness is necessary, moment condition is tight.
Concluding Remarks

- Sharpness characterizes throughput scalability under a moment condition which depends on the topology:
  - moment condition depends on the dimension of the grid,
  - results can be extended to trees, irregular topologies.
- It is strictly stronger than deadlock avoidance condition shown in [Karp et al. JACM67].
- Most of the communication protocol (including TCP) satisfies sharpness condition.
Thank you!

Many thanks to: François Baccelli, Zhen Liu, Anton Riabov.

[Chap.3, *Processes of Interaction in Data Networks*]

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