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Sharpness
a Tight condition for Throughput Scalability

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Tip #1: Motivation

- Application deployed on a large distributed system
  (peer to peer networks, data stream processing, …)
- It is throughput scalable if the rate of computation and communication tasks is independent of the size.

- Challenge: What are the exact conditions of such scalability?

Tip #1: Motivation (cont'd)

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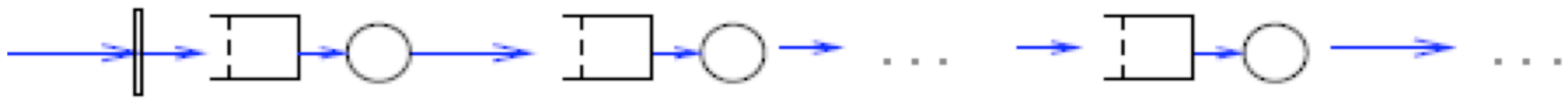
- Infinite tandem of single-server queue



  – Service of customer m starts in station k as soon as m has arrived and k is available:

$$T(m,k) = S(m,k) + \max\left(T(m,k-1), T(m-1,k)\right) .$$

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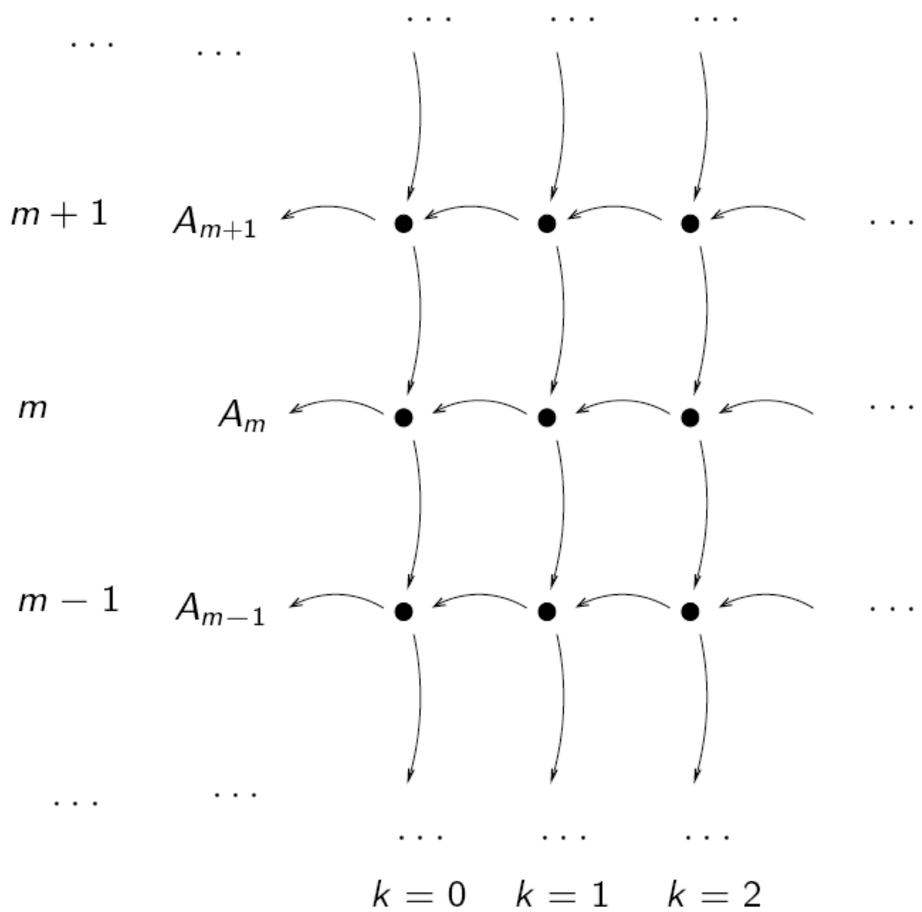
Tip #1: Motivation (cont'd)

- Infinite tandem of single-server queue



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Tip #1: Motivation (cont'd)

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- Infinite tandem of single-server queue
  - with finite buffer and BLOCKING

  - New constraint (in addition to previous one):
    - Task T(m,k) is allowed only when T(m-B,k+1) is done.

  - Throughput scalability still holds [Martin QUESTA 2002]

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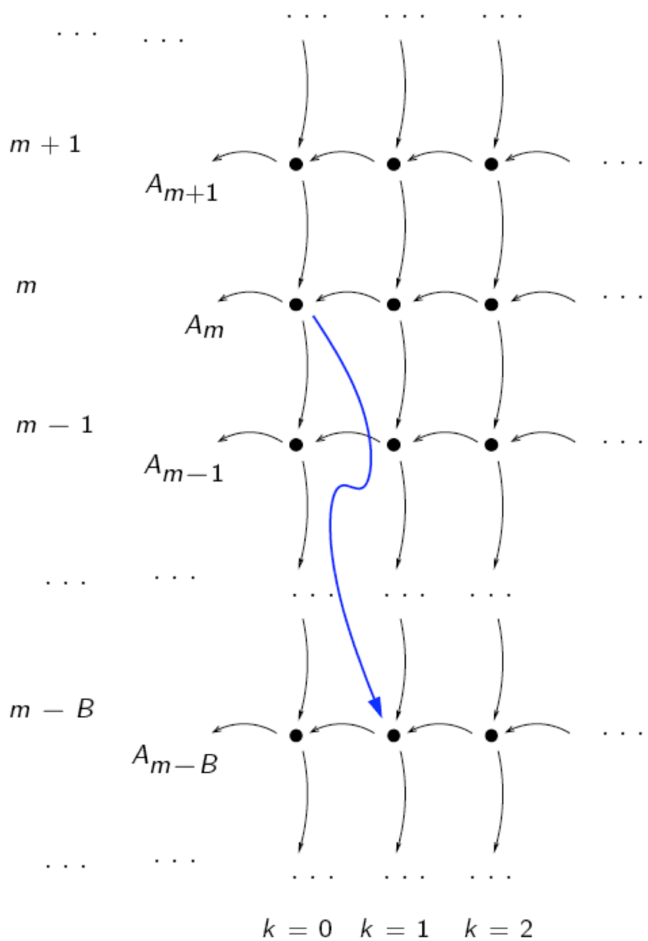
Tip #1: Motivation (cont’d)

- Infinite tandem of single-server queue



Tip #2: The model used in this talk

- A directed graph :
  - vertices represent tasks to complete
  - edges represent dependencies

  Vertices set are $\mathcal{V} = \mathbb{Z}^d \times \mathcal{H}$ where $\mathcal{H}$ is **finite**.

  Collection of Edges $\mathcal{E}$ is **invariant** by translation.

- This model is analogous to *Uniform Recurrence Equations* [Karp et al. JACM67].

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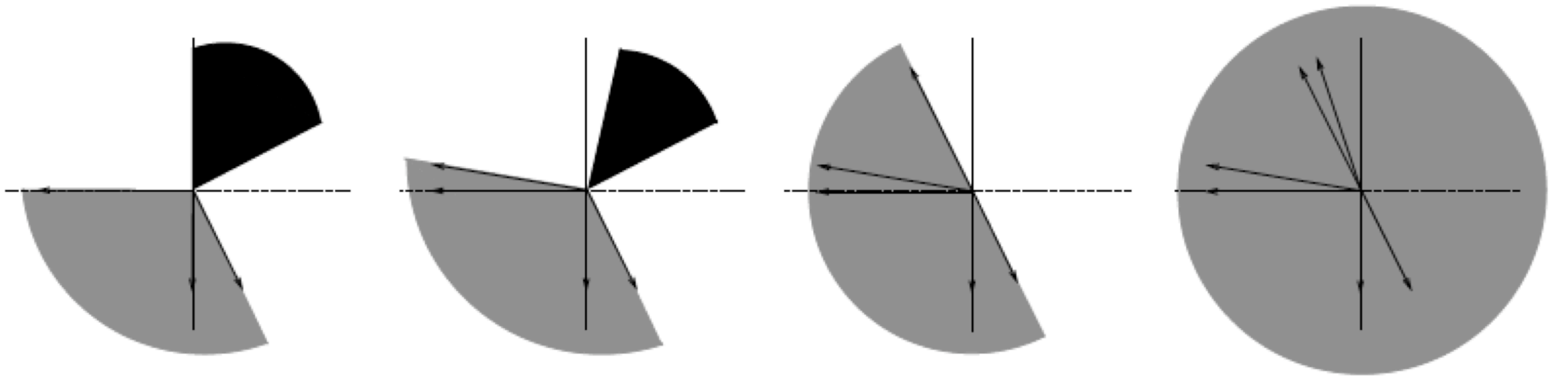
Tip #3: The main result

- Sharpness:
  - a dependence path $\pi : a \times h \rightsquigarrow a' \times h'$ is called a dependence cycle if $h' = h$. We define its associated vector as $r = a' - a$.

CONDITION 1. *There exists s in $\mathbb{Z}^d$, called a* sharp vector, *such that* $<r, s> \leq -1$ *for all r associated with a dependence cycle.*



(a) (b) (c) (d)

Tip #3: The main result (cont'd)

- The system is throughput scalable if
  - the sharpness condition holds,
  - the weights satisfy the moment condition $\int_0^\infty \mathbb{P}\left[\bar{s} \geq u\right]^{\frac{1}{d}} du < \infty .$

Sketch of the proof

- tasks completion time = last-passage percolation time
- sharpness implies that dependence paths have the same combinatorial properties as lattice's connected subsets.

- Sharpness is necessary, moment condition is tight.

Concluding Remarks

- Sharpness characterizes throughput scalability
  (under a moment condition which depends on the topology)
  - moment condition depends on the dimension of the grid,
  - results can be extended to trees, irregular topologies.
- It is strictly stronger than deadlock avoidance condition shown in [Karp et al. JACM67].
- Most of the communication protocol (including TCP) satisfies sharpness condition.

Thank you!

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[Chap.3, *Processes of Interaction in Data Networks*]

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